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## OPTIMIZING METAL CUTTING COST BY INTEGRATION OF COST OF QUALITY USING TAGUCHI'S LOSS FUNCTION

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### ABSTRACT

*The metal cutting is one of the most basic and common operations in manufacturing industries. The quality of final product depends largely upon the control of metal cutting operations performed on its components. There are many characteristics of the machined components which directly contribute to the product quality. These characteristics may include dimensional accuracy, surface finish, roundness, etc. Most of the characteristics of a work-piece are the function of machine-tool and cutting-tool condition, cutting parameters, material of the tool and work-piece, and worker training. The production planners generally don't have control over the material of the work-piece and condition of machine-tool. The worker training problems are addressed separately based upon the availability, time, learning curve issues, etc. However, the cutting parameters are totally controlled by the production planner. Therefore, the selection of the cutting parameters becomes a critical issue to control the quality. The selection of the cutting parameters is mainly dependent on the economics of metal cutting operations. However, traditionally these economic models do not include the quality related cost. Even though, some recent attempts have been made to include quality issues in the tool economics models cutting parameters but the relationship between the surface roughness and cutting parameters has not been explored extensively. This paper is an attempt to develop a model to include surface roughness into the tool-economics model to select cutting parameters using Taguchi's loss function approach.*

### INTRODUCTION

The metal cutting operations are fundamental to most discrete part manufacturing industry. The quality of the products which use parts produced via metal cutting operations largely depends upon quality of metal cutting operations. The quality issues had not received much attention in the manufacturing operations until 1960-1970s. However, increased competitiveness and globalization of economy has created a revolutionary change in the quality practices and management. The concept of quality has also been changing to include every aspect of an organization. These quality driven changes also had impact on metal cutting industry. The production planners have find ways to improve quality of each metal-cutting operation. Therefore, it is becoming essential to consider

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quality related factors in every aspect of the metal-cutting operation. The most fundamental aspect of the metal-cutting operations is the selection of cutting parameters, i.e., cutting speed, feed rate, and depth of cut. The selection of these three main machining parameters is dependent on the cost of the metal cutting operation. The traditional cost models for metal cutting operations mainly included cost of machining, setup and tool. In general, these model models are based on optimization of processing cost (tool cost, machine-use cost, regrinding cost, tool change cost) or tool life subjected to technological constraints such as limits on cutting parameters, machine torque, spindle force, etc. Optimization of a given cost model provides the most economic values of these parameters to operate the machine. But these approaches of metal cutting economics (for examples, Lambert & Walvekar, 1978; Davis, Wysk & Agee, 1978; Maheshwari, 1984; Schall & Chandra, 1990) are simply based on the time and tool factors and do not allow considerations for some other important factors which are directly related to the machining operation.

Two important components ignored in the traditional metal cutting cost models are dimensional accuracy and quality of the surface finish. Mostly, it is assumed that there is no significant change either in the dimensional accuracy or in the surface roughness as a metal cutting operation proceeds before a tool change is necessary. This make the life of cutting tool (time before tool must be changed) as a main driving force in the traditional economic models of metal cutting operations. The variations in the tool life are also extensively studied (Ramalingam & Watson, 1977; 1978) and are generally showed to vary with the basic cutting parameters. However until recently, the relationship between tool life and quality characteristics has not received much attention from researcher. Some recent literature (Robles & Roy, 2004; Wu & Chyu, 2004; Hui, Leung & Linn, 2001; Choi & Park, 2000; Cheng & Saeed, 1995; Hui & Leung, 1994) indicates that quality considerations are becoming more important in the economics of metal cutting. These quality considerations in the selection of cutting parameters along with intelligent monitoring systems can possibly deliver much higher quality at lower cost.

The poor quality of workpiece is largely influenced by the condition of cutting-tool which is a function of cutting parameters (Mittal & Mehta, 1988; Maheshwari, Mishra & Mehta, 1991.) A challenge to the planner is what parameters to select to improve the product quality at minimum overall cost. Both the workpiece and the cutting tools are routinely inspected. Computerized tool and workpiece inspections could be used to enhance monitoring process (Kendall & Bayoumi, 1988.) Based on the condition of the tool or the quality of the parts at a given time, an operator can decide to change the cutting tool. As indicated earlier, the condition of tool depends upon the cutting parameters. Thus, the selection of cutting parameters--cutting speed, feed rate and depth of cut, in a metal cutting operation becomes very critical. However, these models do not include the cost of resulting quality such as dimensional accuracy, surface finish, etc. The literature (Wu & Chyu, 2004; Hui, Leung & Linn, 2001; Cheng & Saeed, 1995; Hui & Leung, 1994) has shown the cost incurred due to deviation in quality of the workpiece during the cutting process is an important component of the tool economics. Taguchi's loss function (Taguchi, 1988) is used to model the cost of the dimensional quality deviations.

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The Taguchi model includes a cost of deviation from preset quality level. It models quality cost as a quadratic loss function. That is, any deviation from the quality results in loss to the society which increases somewhat exponentially as the deviation from the preset quality limit increases. This approach has been applied to a wide variety of fields like from manufacturing to service industry with success; for example computer fraud detection (Schölkopf & Smola, 2002) real estate service (Kethley, Waller & Festervand, 2002) or quality of hog (Roberts, 1994.) In this study, two separate quality dimensions of metal cutting are included. These two dimensions are size (tolerances) and surface quality (roughness). A model for tool economics with the application of Taguchi's quadratic loss function to dimension accuracy and surface roughness is developed below.

### MOTIVATION

The new tool economics model found in the literature normally are limited to only one quality characteristics; dimensional accuracy. However, the quality of the workpiece is defined by several characteristics including surface finish. We believe that the surface finish should be the part of the tool economics model along with dimensional accuracy and other traditional factors. The surface roughness can contribute to the significant losses in certain components such as automobile pistons, cylinders and bearing. Therefore, ignoring the surface finish cost during the machining operations could result in greater losses during the actual usage of the product. However, these losses can be reduced if quality cost due to surface roughness is included in the tool economics model. The cutting parameters can be selected such that the losses due to the surface roughness are reduced.

### MODEL FORMATION

The tool economics model is expanded to include the losses due the surface roughness. Taguchi loss function is used to incorporate surface roughness losses. The other traditional cost factors are also included. The closed form mathematical can be formulated with certain practical assumptions. The model assumptions are listed below:

1. Only flank wear influences the dimensional accuracy and surface finish and other factors have no significant impact on the surface finish and dimensional accuracy.
2. Tool life is function of the flank wear only.
3. The quality loss can be defined using Taguchi functions.
4. The catastrophic failure of the tool is ignored.
5. Tool- life can be restored completely after regrinding.
6. The mean rate of tool wear and standard deviation of tool wear is known.

**Notations**

The following notations are used in the model formulation:

V	Cutting speed.
d	Depth of cut.
f	Feed rate.
r	Rate of tool wear.
H	Cumulative height of flank wear.
h	Height of tool wear at a give time, t.
t	Time instance during a machining operation.
T	Total time.
L	Tool life.
$x_1$	Random variable, dimensional accuracy.
$x_2$	Random variable, surface roughness.
m	Desired value of a dimension of a component.
$l(y)$	Loss due to the deviation from the desired value.
y	Average dimension produced by the machining processes.
$\sigma_1$	Standard deviation of dimensional accuracy.
$\sigma_2$	Standard deviation of surface roughness.
m	Desired value of a dimension.
$R_a$	Average roughness.
K	Tool life equation constant.
$K_{\text{lossi}}$	Constant associated with Taguchi loss function.
$n_j$	Exponent associated with tool life.
a, b, c	Constants, cost factors.

**TAYLOR'S TOOL LIFE MODEL**

The tool-economics models largely depends upon the Taylor's tool life equation. This equation relates the tool life (L) with the cutting speed and described as below:

$$VL^{n^*} = \text{Constant}$$

The above equation can be rewritten as:

$$L = K V^n$$

The equation shows that the tool life is a function of the cutting speed only. The constant  $K$  depends upon material of tool and workpiece. This above equation has been modified to reflect the impact of other machining parameters feed rate and depth of cut. The modified Taylor's tool life equation is:

$$L = K V^{n_1} f^{n_2} d^{n_3}$$

The literature shows other factors, such as machine torque, may also influence tool life. However, in this paper will restrict modeling to the three important cutting parameters, speed, feed and depth of cut.

As per our assumption (6), the tool life is a function of flank wear only. The above equation can be related to the height of flank wear. In most of the cutting operations, the flank wear has far more significant impact on the tool life than any other kind of wear. Hence, our assumption is very realistic and can be used without loss of any generality. The above equation can be modified as:

$$\begin{aligned} L &\propto H \\ L &= K_1 * H \end{aligned}$$

It is also well known in the metal cutting literature that the tool-wear rate is somewhat constant during the most of the useful life of a tool. Therefore, the cumulative flank wear can be related to the rate of tool wear as:

$$h = vt$$

Above equation is valid only during the constant wear rate zone. The initial increasing wear rate zone is ignored, as that period is normally very small compared to the constant wear rate zone. The same is true for the last zone as tool deteriorates fast and is removed from the operations for regrinding or exchange.

### TRADITIONAL TOOL ECONOMICS MODEL

The traditional tool-economics models largely depend upon the Taylor tool life. These models include the setup cost, machining cost and tool cost. A per unit cutting cost model can be written as:

$$\begin{aligned} \text{Cutting cost} &= \text{machining cost} + \text{tool cost} + \text{set up cost} \\ &= \frac{a}{V} + bV^{(n_1-1)} + c, \end{aligned}$$

If use modified Taylor's equation:

$$= aV + bV^{(n1-1)} f^{n2} d^{n3} + c.$$

## QUALITY COST MODEL

### Dimensional Accuracy

A machined part is considered acceptable as long as it's dimensions are within the tolerance limits. However, Taguchi's loss function approach assumes there are losses for any deviation from the desired value. The desired value is normally the design dimension of the component. However due to the manufacturing limitations, some deviations from the designed value is acceptable. The acceptable deviation limits are called tolerances. According to the loss function approach, even the deviation within the tolerance limits results in the losses to the larger society. These losses are represented as:

$$l(y) = K_{\text{loss}} (y-m)^2$$

Due to tool wear at the flank, the cutting conditions are changing continuously. The depth of cut is reduced by the amount of the tool wear at the flank. Hence, the dimensional accuracy is changing along with the tool wear. Furthermore, it is also well documented in the literature that the tool wear is a stochastic phenomenon. Therefore, there is some randomness in the amount of tool wear along with a deterministic wear component. The random component of the tool-wear will also result in the dimensional deviations hence will result towards the loss. Total deviation from the actual dimension at any given time can be written as:

$$(\text{Deterministic component} + \text{random component of dimensional changes}) = (tt + x_1)$$

Using Taguchi's loss function, at any given time "t", the losses due to this change can be formulated as:

$$l(y) = K_{\text{loss}} (tt+x_1 - m)^2;$$

Assume that mean of the desired variation is zero.

$$m = 0;$$

Hence;

$$\begin{aligned}
 \text{Total loss due to the dimensional accuracy at time 't'} &= K_{1\text{obs}}(rt + x_1)^2 \\
 \text{Expected loss due to the dimensional accuracy at time 't'} &= E[K_{1\text{obs}}(rt + x_1)^2] \\
 &= E[K_{1\text{obs}}(r^2t^2 + x_1^2 + 2rt x_1)] \\
 &= K_{1\text{obs}}(r^2t^2 + \sigma_1^2 + 2rt m) \\
 m &= 0;
 \end{aligned}$$

$$\begin{aligned}
 \text{Expected loss due to the dimensional accuracy at time 't'} &= K_{1\text{obs}}[r^2t^2 + \sigma_1^2] \\
 \text{Total loss due to the dimensional accuracy over time T} &= \int_0^T K_{1\text{obs}}(r^2t^2 + \sigma_1^2) dt \\
 &= K_{1\text{obs}}(r^2T^3/3 + \sigma_1^2 T) \\
 \text{Mean loss over time T} &= K_{1\text{obs}}(r^2T^2/3 + \sigma_1^2)
 \end{aligned}$$

Using:

$$\begin{aligned}
 rT &= H \\
 \text{Mean losses due to the dimensional accuracy} &= K_{1\text{obs}}[\sigma_1^2 + (1/3) H^2]
 \end{aligned}$$

## Surface Finish

The surface finish is measured in terms of the roughness value of the surface. Lower the value of surface roughness better it is for the quality. Hence, any deviation in the roughness value on the negative side does not result in any loss. The literature on the machining operations shows that the tool wear and surface finish are closely related. The relationship between flank wear and roughness value can be formulated as:

$$\begin{aligned}
 R_s &\propto h^2 \\
 R_s &= K_2 h^2
 \end{aligned}$$

However, all of the variations in the surface cannot be explained alone by the tool wear height. Assuming that all other variations in the roughness are due to random causes, the total roughness can be written as:

$$R_s = (x_2 + K_2 h^2)$$

Assuming that the mean desired value of roughness is zero, meaning by completely smooth surface. The losses due to the surface roughness can be modeled using Taguchi's loss function in a similar way as the formulation of the dimensional accuracy losses. The losses can be formulated as:

$$\begin{aligned}
 \text{Losses due to surface roughness at a given time 't'} &= K_{1\text{obs}}(x_2 + K_2 h^2 - \text{mean desired value})^2 \\
 &= K_{1\text{obs}}(x_2 + K_2 (rt))^2 \\
 &[\text{mean desired value of roughness} = 0]
 \end{aligned}$$

$$\begin{aligned}
\text{Expected losses due to roughness at a given time 't'} &= E[K_{10\text{as}2}(x_2 + K_2 (rt)^2)^2] \\
&= K_{10\text{as}2} E[x_2^2 + K_{10\text{as}2}^2 (K_2)^2 (rt)^{2\alpha}] \\
&\quad + K_{10\text{as}2} E[K_2 (rt)^2 x_2] \\
&= K_{10\text{as}2} \sigma_2^2 + K_{10\text{as}2}^2 (K_2)^2 (rt)^{2\alpha} \\
&\quad + K_{10\text{as}2} [K_2 (rt)^2 * \text{mean}] \\
&\quad \text{(Assuming mean of } x_2 \text{ is zero).} \\
&= K_{10\text{as}2} \sigma_2^2 + K_{10\text{as}2}^2 (K_2)^2 (rt)^{2\alpha} \\
\text{Total losses due to roughness in Time T} &= \int_0^T \{K_{10\text{as}2} \sigma_2^2 + K_{10\text{as}2}^2 (K_2)^2 (rt)^{2\alpha}\} dt \\
&= K_{10\text{as}2} \sigma_2^2 T + (1/(2s+1))K_{10\text{as}2}^2 (K_2)^2 (rT)^{2s+1} \\
\text{Mean losses due to the roughness} &= K_{10\text{as}2} \sigma_2^2 + (1/(2s+1))K_{10\text{as}2}^2 (K_2)^2 (rT)^{2s}
\end{aligned}$$

Using:

$$rT = H$$

$$\begin{aligned}
\text{Mean losses due to the surface roughness} &= K_{10\text{as}2} \sigma_2^2 + (1/(2s+1))K_{10\text{as}2}^2 (K_2)^2 (H)^{2s} \\
\text{Mean total losses due to quality} &= K_{10\text{as}1} [\sigma_1^2 + (1/3) H^2] + K_{10\text{as}2} [\sigma_2^2 \\
&\quad + (1/(2s+1))(K_2)^2 (H)^{2s}] \\
\text{Per unit loss due to variation} &= (\text{mean losses/unit time})/V \\
&= \{K_{10\text{as}1} [\sigma_1^2 + (1/3) H^2] \\
&\quad + K_{10\text{as}2} [\sigma_2^2 + (1/(2s+1))(K_2)^2 (H)^{2s}]\}/V. \\
\text{Total Cost Function:} & \\
\text{Traditional machining cost + losses due to quality} &= a/V + bV^{(n1-1)} f^{n2} d^{n3}; + c + \\
&\quad \{K_{10\text{as}1} [\sigma_1^2 + (1/3) H^2] \\
&\quad + K_{10\text{as}2} [\sigma_2^2 + (1/(2s+1))(K_2)^2 (H)^{2s}]\}/V.
\end{aligned}$$

## CONCLUSIONS

The paper shows a new formulation of tool-economics using Taguchi function, which includes the quality losses due to dimensional accuracy and surface roughness. The cutting parameters selected using this model will optimize tool cost along with cost incurred due dimensional deviations and surface roughness. It must be noted that by inclusion of Taguchi loss-function in the tool economic model, the cost considerations are not limited just to the accounting considerations of direct labor and material costs but it includes perceived cost in the form of loss incurred upon society due to any variation. This creates a need to calculate of the societal loss constants like  $K_{10\text{as}1}$  and  $K_{10\text{as}2}$ . Further work is needed to estimate such loss constants.



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